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Irreversible Adiabatic Demagnetization: Entropy and Discrimination of a Model Stochastic Process

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A recently proposed evaluation of irreversible entropy changes with the help of model stochastic processes is applied to the demagnetization of an isolated square Ising lattice. A stochastic process (executed with a computer) causes an initially magnetized lattice to undergo an adiabatic demagnetization, with no external work, by flipping the spin's orientation. The condition interaction energy E = const is maintained by a pairing of flips for which δE cancel out mutually. The demagnetization rate is controlled by letting a parameter X^s (which favours one orientation over the other) decrease toward zero at the desired rate; the rate ranges from an effectively reversible to the most irreversible demagnetization. The external entropy change is evaluated from the process *discrimination*, related to the transition probabilities of the actual steps. Together with the state entropy of the lattice, this enables one to find the net entropy production characterizing an irreversible process. The evaluation is achieved without recourse to thermodynamic equivalents. A thermodynamic description, in terms of an equivalent temperature for the adiabatic process, is presented separately.

KEY WORDS: Irreversible processes; stochastic models; computer simulation; entropy production; discrimination; Ising lattice; adiabatic demagnetization; equivalent temperature.

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1. INTRODUCTION

A recent article[1] discussed the entropy of model stochastic processes. Suppose that a nonisolated N-particle system undergoes an arbitrary variation. The associated entropy changes are conveniently written as[2]

$$\delta\phi = \delta S_{\text{syst}} - \delta S_{\text{ext}} \geqslant 0 \tag{1}$$

Here δS_{syst} (or δS) is the entropy difference between the final and initial states of the system, while δS_{ext} is the flow of entropy into the system due to the action of surroundings (external constraints). The net entropy production $\delta\phi$ (also called internal entropy change) equals zero in a reversible variation; in an irreversible variation it is larger than zero. This depends on whether or not the system's response δS matches the action of the constraints δS_{ext}. The evaluation of $\delta\phi$ is of considerable interest, both as an indicator of irreversibility and in connection with the principle ascribing pathways of minimum entropy production to incompletely specified processes.

Suppose further that our variation is described with the help of a stepwise stochastic process. At time s of the process the configurations (microstates) of the system transform one into another in a sequence of individual particle transitions $s, s+1, s+2,..., s+N$. Can $\delta\phi$ be evaluated for this sequence of steps? Statistical mechanics relates the entropy of a system to the probability distribution of its configurations, or, specifically, to the average value of $-\log p_i$, p_i being the probability of the ith configuration. The probabilities p_i are defined for any instant of a stochastic process and hence (at least in principle) δS can be evaluated from

$$\delta S = -\delta\langle\log p\rangle = \langle\log p\rangle_s - \langle\log p\rangle_{s+N} \tag{2}$$

In order to evaluate δS_{ext} we turn to the sequence of the actual transitions. Thus $f_{s,s+1}$ is the probability of the transition which has occurred at step s of the process, while $f_{s+1,s}$ is the probability of the reverse transition. Their ratio measures the degree to which the former is favored over the latter. To make the quantity extensive in N, the total *discrimination* of a particular execution of the process is defined by

$$\delta D = \sum_{s'=s}^{s+N} \log(f_{s',s'+1}/f_{s'+1,s'}) \tag{3}$$

The discrimination expresses the stochastic effort with which the process is driven. By contrast, $-\delta\langle\log p\rangle = \delta S$ represents the corresponding response in the form of a changing probability distribution. This suggests the identification of δD, or, better, of its average over repeated executions of the process, with the entropy change due to the action of constraints. Thus the basic postulate of Ref. 1 is

$$-\delta S_{\text{ext}} = \langle\delta D\rangle \tag{4}$$

$$\simeq (\delta D)_{\text{observable}} \qquad \text{for large } N \tag{4'}$$

Equations (1), (2), and (4) combined enable one to express $\delta\phi$ for a stochastic process

$$\delta\phi = -\delta\langle\log p\rangle + \langle\delta D\rangle \tag{5}$$

The following evidence was presented in Ref. 1 to support the postulate. (1) An examination of the dynamics of a stochastic process for both equilibrium and nonequilibrium shows that $\langle\delta D\rangle$ is, respectively, equal to, and larger than, $\delta\langle\log p\rangle$. This agrees with $\delta\phi \geqslant 0$ as required by the second law of thermodynamics. (2) This agreement has been corroborated by actual computer experiments, which simulated the cooling of a square Ising lattice in contact with a reservoir. Two manners of cooling were considered: First, individual spins exchange heat with the reservoir, and second, the lattice cools as a bulk system, both through a heat exchange with the reservoir and by an internal redistribution of the absorbed heat. A "Metropolis"-like[(3)2] Monte Carlo process describes the first manner of cooling; an approximate description of the second was introduced in Ref. 1 and dubbed the "cooperative model." Both models were executed with the help of a computer. The total entropy production ϕ calculated with Eq. (5) was invariably positive and decreased to zero as the duration of the cooling increased. Another reasonable result was that $-\phi$ could be decreased by improving the internal equilibration mechanism of the cooperative model. (3) Whenever $-\delta S_{\text{ext}}$ can be expressed in terms of thermodynamic variables, $\delta\langle D\rangle$ should automatically reduce to that expression. This was shown to be trivially true for the Metropolis model, when the transitions at each step obey

$$\delta D = \log(f_{s,s+1}/f_{s+1,s}) \equiv -\beta_{\text{ext}}\,\delta E_{s,s+1} \tag{6}$$

β_{ext} being the reciprocal temperature of the reservoir and $\delta E_{s,s+1}$ the energy change for the transition. However, to an exchange of the heat $\delta Q = \delta E_{s,s+1}$, there corresponds

$$-\delta S_{\text{ext}} = -\beta_{\text{ext}}\,\delta Q = -\beta_{\text{ext}}\,\delta E_{s,s+1} \tag{7}$$

A more indirect argument, relying on a concept of an equivalent temperature of the joint external and internal heat exchange, showed this to be still true for the cooperative model. In that case, however, a physically meaningful temperature can be defined only for configurational transitions which involve a large number of stochastic steps, viz. for large N.

The last point of the foregoing discussion, concerning the equivalence to thermodynamic expressions, makes one wonder whether the attempt to introduce discrimination as a separate concept is not rather superfluous. Why not express the ratios $f_{s,s+1}/f_{s+1,s}$ by their equivalent thermodynamic variables and thence calculate δS_{ext} for a process with the help of standard irreversible

[2] See Ref. 4 for application to the Ising lattice.

thermodynamics? Our answer is that the discrimination remains evaluable for a process described in a consistently probabilistic manner (by transition probabilities), even when the definition of equivalent thermodynamic variables becomes artificial or even doubtful. The present article tries to substantiate this claim by presenting a calculation of the discrimination and of ϕ for a stochastic process describing a demagnetization with no work, which proceeds irreversibly and adiabatically, so that the temperature is ill defined. (In other words, we try, following Brillouin, to "exorcise Maxwell's Demon"[5] by evaluating the *net* entropy production for a process, preferring, however, to perform the requisite rites in a consistently probabilistic language.)

2. THE MODEL STOCHASTIC PROCESS AND ITS INTERPRETATION

Consider a thermally isolated square Ising lattice, consisting of N spins. The lattice is initially magnetized and is allowed to demagnetize during the process, either abruptly or in a gradually controlled manner; an example of a subsequent remagnetization is considered as well. The initially magnetized lattice is constructed as follows. In a succession of steps $s = -N + 1$, $-N + 2$, $-N + 3, \ldots, 0$, the N spins are accorded a magnetic orientation $\sigma_s = 1$ or -1. The probabilities at each step are

$$f^0(\sigma_s) = \exp(X^0\sigma_s)/2\cosh(X^0\sigma_s) \tag{8}$$

Here the adjustable parameter $X^0 > 0$ achieves the net magnetization of the lattice by making the probability for $\sigma_s = 1$ larger than the probability for $\sigma_s = -1$, viz. $f^0(+) > f^0(-)$. Thus the execution (with the aid of a computer) of a particular N-step process gives the following net magnetization (per spin):

$$M^0 = N^{-1} \sum_{s=-N+1}^{0} \sigma_s \simeq f^0(+) - f^0(-) = \tanh(X^0) \tag{9}$$

where the second, approximate equality neglects fluctuations for sufficiently large N. The internal energy per spin due to the interaction of each spin with its four lattice neighbors is

$$\begin{aligned} -E = (2N)^{-1} \sum_{s=-N+1}^{0} \sum_{s'=1}^{4} \sigma_s\sigma_{s'} &\simeq 2[f^0(+) - f^0(-)]^2 \\ &= 2(\tanh X^0)^2 \end{aligned} \tag{10}$$

Another important quantity is calculated from the product of the probabilities $f^0(\sigma_s)$ for the sequence of the actual steps, $s = -N + 1$ to $s = 0$. This gives the probability p^0 of a particular stochastic construction, or equivalently, of an initial lattice configuration. With neglect of fluctuations for large N, $\log p^0$ can be identified with the average value. This enables one to calculate the entropy of the probability distribution initially accorded to the system's configurations [see Eq. (2) or elsewhere[6]],

$$\sum_{s=-N+1}^{0} \log f^0(\sigma_s) \simeq \langle \log p^0 \rangle = -S^0 \qquad \text{for large } N \tag{11}$$

After completion of the initial construction the lattice is allowed to lose its initial magnetization without performing work and adiabatically, viz. while keeping E constant. The appropriate stochastic process proceeds in a sequence of steps $s = 1, 2, 3, \ldots, \omega\ (> N)$. At each step two nonadjacent spins of equal orientation are flipped jointly, so that $\delta M_s = 4$ or -4. In order to ensure that $\delta E_s = 0$, the joint flips must belong to one of the following six groups of allowed transitions:

(a) Both flipped spins have $\sigma_s = -1$, so that $\delta M_s = 4$ and the four neighbors to the first and the second spins are, respectively, $4(+)$ and $4(-)$:

$$\begin{array}{ccc} & + & \\ + & - & + \\ & + & \end{array} \quad \text{and} \quad \begin{array}{ccc} & - & \\ - & - & - \\ & - & \end{array}$$

(b) Same as (a) but the flipped spins have $\sigma_s = 1$, so that $\delta M_s = -4$:

$$\begin{array}{ccc} & + & \\ + & + & + \\ & + & \end{array} \quad \text{and} \quad \begin{array}{ccc} & - & \\ - & + & - \\ & - & \end{array}$$

(c) Both flipped spins have $\sigma_s = -1$ and the four neighbors to the first and to the second spins are, respectively, $3(+) + 1(-)$ and $1(+) + 3(-)$:

$$\begin{array}{ccc} & - & \\ + & - & + \\ & + & \end{array} \quad \text{and} \quad \begin{array}{ccc} & + & \\ - & - & - \\ & - & \end{array}$$

(d) Same as (c) but $\delta M_s = -4$:

$$\begin{array}{ccc} & - & \\ + & + & + \\ & + & \end{array} \quad \text{and} \quad \begin{array}{ccc} & + & \\ - & + & - \\ & - & \end{array}$$

(e) Both flipped spins have $\sigma_s = -1$ and the four neighbors to the first and to the second spins alike are $2(+) + 2(-)$:

$$\begin{array}{ccc} & - & \\ - & - & + \\ & + & \end{array} \quad \text{and} \quad \begin{array}{ccc} & - & \\ - & - & + \\ & + & \end{array}$$

(f) Same as (e) but $\delta M_s = -4$:

$$\begin{array}{c} - \\ -++ \\ + \end{array} \quad \text{and} \quad \begin{array}{c} - \\ -++ \\ + \end{array}$$

In practice the process is executed as follows. All lattice spins are currently classified according to their four neighbors, which fixes their adherence to one of the above groups of candidate joint flips. The actual pair flipped at step s is determined with the help of three Monte Carlo lotteries. First lottery determines the group; the second and the third lotteries determine, by random choice, which of the first and second candidate spins belonging to the group are actually flipped (whereupon all groups are updated). In the first lottery a weight factor $F^s(i)$ is accorded to each of the groups $i = 1. 2,..., 6$. The factor is proportional to the numbers of the first and second candidate spins, $n_i^s \times m_i^s$, and depends on whether δM_i is equal to 4 or to -4. Thus

$$F^s(i) = (\Omega^s)^{-1} n_i^s m_i^s \exp(X^s\, \delta M_i/2) \tag{12}$$

Ω^s being the normalization coefficient summing the weight factors of the six groups at time s. The adjustable parameter $X^s \leqslant X^0$ controls the decrease of the magnetization with increasing s. $X^s \equiv X^0$ would preserve the initial magnetization [see Eq. (12) versus Eq. (8) applied to $2 \times \sigma_s$]. $X^s \equiv 0$ corresponds to the most abrupt demagnetization. A controlled demagnetization is obtained by allowing X^s to decrease gradually from the initial value X^0 to the ultimate value $X^s = 0$.

The actual execution of a model process enables one to measure the net lattice magnetization M and the discrimination as functions of the time. The calculation of the discrimination for step s is as follows. The first lottery proceeds with the probability $F^s(i)$, the second and third lotteries (picking the actual spins out of the group) proceed with the probabilities $1/n_i^s$ and $1/m_i^s$, respectively. The transition probability of an s step therefore is

$$f_{s,s+1} = F^s(i)/(n_i^s m_i^s) = (\Omega^s)^{-1} \exp(X^s\, \delta M_s/2) \tag{13}$$

where δM_i for step s is denoted by δM_s. The discrimination of N steps of the process therefore is [cf. Eq. (3)]

$$\delta D = \sum_{s'=s}^{s+N} X^{s'}\, \delta M_{s'} \simeq \langle \delta D \rangle \qquad \text{for large } N \tag{14}$$

Suppose the decrease of X^s with s is gradual enough to make the process effectively reversible, $\delta\phi \simeq 0$. In that case [cf. Eqs. (2) and (5)]

$$\delta D \simeq \delta\langle \log p \rangle = -\delta S \qquad \text{(reversible variation)} \tag{15}$$

This enables us to calculate the entropy of the system at the end of the process $s = 1, 2, 3, \ldots, \omega\ (> N)$ from

$$S = S^0 + \Delta S \simeq S^0 - D \qquad \text{(reversible variation)} \tag{16}$$

with S^0 and D evaluated with the help of Eqs. (11) and (14), respectively. Having found S for the reversible process enables one to compute the entropy production [Eq. (5)] for an irreversible process which converges upon the same terminal state at ω

$$\phi = S - S^0 + D \tag{17}$$

But at the end of the process $X^s = 0$ and the exchange of spin orientation is precisely like that for an Ising lattice at equilibrium, with the (invariant) lattice energy known from Eq. (10). This serves to fix the equilibrium reciprocal temperature $\beta_{\rm eq}$ and the corresponding $S(\beta_{\rm eq})$ with the help of Onsager's theory.[3] This theoretical $S(\beta_{\rm ext})$ can be compared to the value found with the help of Eq. (16).

Can the results be related to thermodynamic quantities, before the process converges toward $X^s = 0$? A well-known approximation[(8)] treats a reversible "essentially" adiabatic process as if proceeding in thermal contact with surroundings at $\beta_{\rm ext}$. Accordingly, our magnetizing parameter X^s can be interpreted as a Boltzmann exponential coefficient due to a magnetic field H acting on a system at $\beta_{\rm ext}$

$$X^s = \beta_{\rm ext} H \tag{18}$$

so that Eqs. (13) and (18) combined reproduce the detailed balance equation for two configurations differing by the energy $-H\,\delta M_s$

$$f_{s,s+1}/f_{s+1,s} = \exp(\beta_{\rm ext} H\,\delta M_s) \tag{19}$$

The interpretation of X^s as a product of $\beta_{\rm ext}$ and H, and the observation that at equilibrium the lattice (of constant volume) is described by two of the three variables E, $\beta_{\rm ext}$ and H, jointly imply that the instantaneous thermodynamic state of the system is fixed by stipulating given X^s, constant E, and an almost reversible variation. Yet the actual decomposition of X^s into $\beta_{\rm ext}$ and H cannot be achieved with the help of Onsager's theory, since this does not hold for a nonzero magnetic field. However, $\beta_{\rm ext}$ can be evaluated with the help of an internally measured reciprocal temperature $\beta_{\rm int}$ in the following way. Take

[3] Reviewed, e.g., in Ref. 7.

a spin together with its four neighbors $j = 1, 2, 3$, and 4, denoting by q the sum of the neighbors' orientations

$$q = \sum_{j=1}^{4} \sigma_j \tag{20}$$

Since the orientation of the central spin may be either σ or $-\sigma$, a pair of conjugate spin states l and l' is defined by

$$l \leftrightarrow (q, \sigma) \quad \text{and} \quad l' \leftrightarrow (q, -\sigma) \tag{21}$$

At any time all spins belong to either one of the five distinct conjugated l, l' pairs, depending on whether $q = 4, 2, 0, -2$, or -4. Let m_l and $m_{l'}$ denote the number of spins which belong to l and l', respectively, while $f_{l,l'}$ and $f_{l',l}$ are the corresponding transition probabilities. At equilibrium

$$m_l/m_{l'} = f_{l',l}/f_{l,l'} \tag{22}$$

If the detailed balance of the two states is supposed to be instantaneously governed by the magnetizing parameter X^s and by a reciprocal temperature β_{int}, then

$$f_{l',l}/f_{l,l'} = \exp(X^s \, \delta M_{l',l} - \beta_{\text{int}} \, \delta E_{l',l}) \tag{23}$$

Equations (22) and (23), together with $\delta M_{l',l} = 2\sigma$ and $\delta E_{l',l} = -2\sigma q$, give

$$\beta_{\text{int}} = (2\sigma q)^{-1} \log(m_l/m_{l'}) - q^{-1} X^s = \beta_{\text{int}}(l, l') \tag{24}$$

in the absence of an internal equilibrium the five conjugated pairs may yield differing values of $\beta_{\text{int}}(l, l')$, which seems to invalidate the thermodynamical significance of this quantity. But for a reversible process, which proceeds at equilibrium,

$$\beta_{\text{int}}(l, l') = \beta_{\text{int}} \qquad \text{for all} \quad l, l' \tag{25}$$

In this case β_{int} measures the equivalent surroundings' temperature for a reversible adiabatic process, which was introduced in Eq. (18),

$$\beta_{\text{int}}(\text{reversible}) = \beta_{\text{ext}} \tag{26}$$

Summing up, Eqs. (18) and (24)–(26) combined permit us to relate X^s (at given E) to the equilibrium values of β_{ext} and H, separately. One may venture that such values of $\beta_{\text{ext}}(X^s, E)$, computed from the reversible process, can be still derived from processes which are moderately irreversible. Although Eqs. (25)–(26) no longer hold, one would say that an irreversible process, at the same X^s and E, *relaxes* toward $\beta_{\text{ext}}(X^s, E)$ under the action of $H = X^s/\beta_{\text{ext}}(X^s, E)$. (A process which converges upon an equilibrium distribution corresponding to β_{ext} and H does so with transition probabilities obeying Eq. (19), in complete equivalence to the Metropolis method.[3,4] In view of the tenuous nature of such remarks, it is stressed that the discussion of

Eqs. (18)–(19) and of Eqs. (22)–(26) relates to an attempted formulation of the thermodynamic equivalents which may, or may not, be valid. Our probabilistic discussion of the stochastic process is summed up by Eqs. (14)–(17) and it will be noted that it is self-contained, requiring no recourse to thermodynamic equivalents.

3. COMPUTER RESULTS

The computer experiments describe the demagnetization of a square Ising lattice of $N = 100 \times 100$ and for internal energy per spin $E \simeq -1.0$. To obtain this value, the construction of the initial lattice was carried out with a magnetization parameter $X^0 = 0.881$ [see the last equality of Eq. (10)]. With a particular sequence of random numbers this gave a starting lattice of $E = -0.995$, initial magnetization $M^0 = 0.704$, and initial entropy $S^0 = 0.419$ [cf. Eqs. (9)–(11)]. The lattice was allowed to demagnetize through the double-flip exchanges $2(+) \rightleftarrows 2(-)$, while E was kept constant [as explained in the paragraph that follows Eq. (11)]. A very gradual demagnetization was achieved by letting the magnetization parameter X^s decrease linearly with the process time s (in jumps of N steps for the sake of convenience) from the initial $X^s = X^0$ to $X^s = 0$ after $s = 146N$ double-flips. The lattice magnetization M, the process discrimination D [Eqs. (14)], and the internal temperature β_{int} [Eq. (24)] were measured over the duration of $s = 170N$ double-flips. The results are described in Fig. 1. The initial value of β_{int} is zero, corresponding to the complete lack of ordering by the initial construction. The subsequent free adiabatic demagnetization induces ordering, to keep E constant, or, equivalently, the lattice cools as M decreases to zero. (Our magnetic cooling can be likened to the Joule effect for a freely expanding, imperfect gas; it vanishes likewise in the absence of strong particle interactions. The question of whether such a demagnetization is experimentally attainable, especially when the rate is controlled by X^s, is not dealt with here.) One notes that when $X^s = 0$ is attained, the values of β_{int} and of $S^0 - D$ [Eq. (16)] are equal to the theoretical values of $\beta_{\text{eq}} = 0.378$ and $S_{\text{eq}} = 0.48$, respectively (calculated with the help of Onsager's theory for an infinite lattice at $H = 0$ and $E = -0.995$). This indicates that the process has indeed proceeded in an essentially reversible manner. Hence the lines for β_{int}, X^s, and M in Fig. 1 describe the *equilibrium* interdependence of $\beta_{\text{int}} \equiv \beta_{\text{ext}}$ [Eq. (26)], $X^s = \beta_{\text{ext}} H$ [Eq. (18)], and M, for an Ising lattice having $E = -0.995$. It is observed that the decrease of M becomes increasingly steep as $M = 0$ is approached, which is expected theoretically (with a phase transition around $M = 0$ occurring when $H = 0$). It is also interesting to note that the entropy change due to the demagnetization at constant E is relatively small ($\Delta S = 0.06$); viz. it does not make much of a difference whether a certain alignment of neighbor spins is

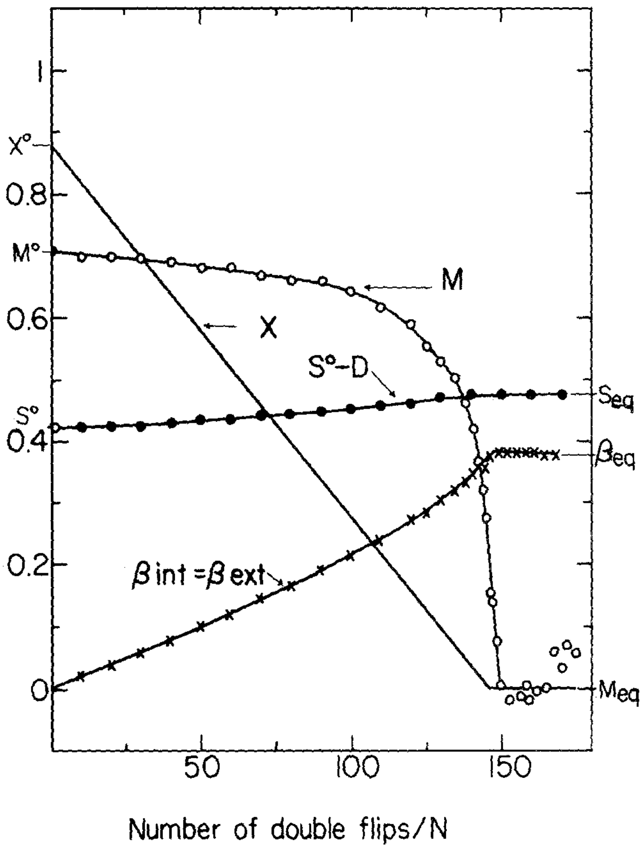

Fig. 1. The reversible adiabatic demagnetization of a square Ising lattice $N = 100 \times 100$ of an internal energy (per spin) $E \simeq -1$. The magnetizing parameter X^s decreases linearly from X^0 to zero over the time interval of $146N$ double-flips; the lattice magnetization M, the entropy change $S^0 - D$ [Eq. (16)], and the internal reciprocal temperature $\beta_{\text{int}} = \beta_{\text{ext}}$ [Eqs. (24)–(26)] are plotted versus the time.

arrived at with the help of an external agency or by means of a spontaneous internal equilibration. The evaluation of $\beta_{\text{ext}} = \beta_{\text{int}}$ for the reversible process makes possible the computation of H from X^s with Eq. (18). The values of H thus computed and the corresponding values of M from Fig. 1 are plotted in Fig. 2; the area under the curve describes the corresponding magnetic energy, $-\sum H\,\delta M$.

Figure 3 describes a fairly abrupt demagnetization in which X^s is allowed to decrease linearly toward zero during the time interval $s = 10N$. We note that $S^0 - D$ at the end of the irreversible process is significantly smaller than S_{eq}, due to the nonzero entropy production $\phi = 0.015$. Yet the thermal

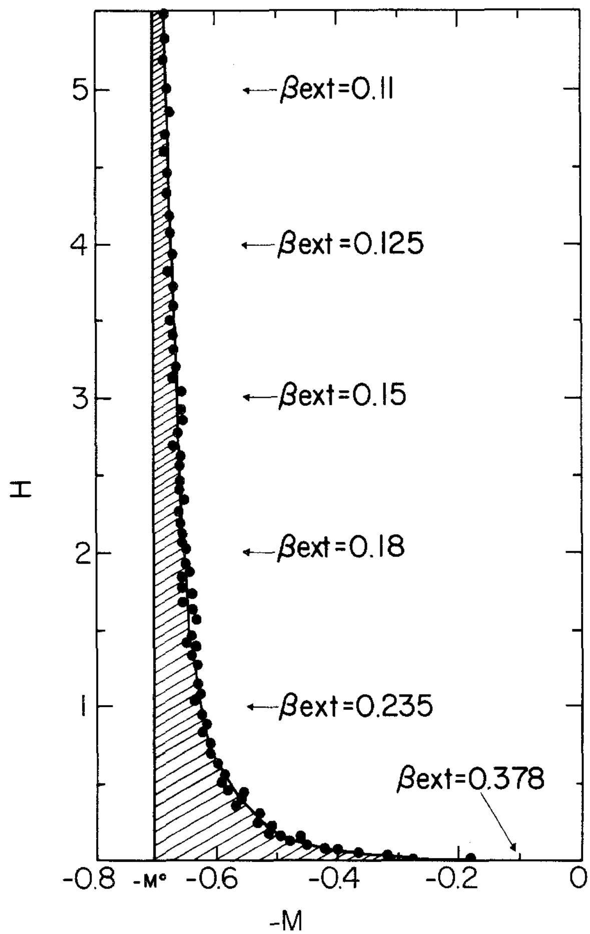

Fig. 2. The "equivalent" magnetic field $H = X^s/\beta_{ext}$ [Eq. (18)] versus the lattice magnetization $(-M)$ for the reversible process of Fig. 1. Area under the curve describes the corresponding magnetic energy.

equilibrium throughout the process is not greatly perturbed. This is indicated by the fact that the values of β_{int} were found to be quite well defined [in the sense of Eq. (25)] and that their dependence on X^s is almost the same as that described by the reversible line $\beta_{int} = \beta_{ext}$ in Fig. 1. This lends experimental support to the hypothesis stated at the end of the preceding section: A moderately irreversible adiabatic process which at time s is specified by X^s and by E, can be interpreted as relaxing in contact with surroundings at β_{ext} and H in accordance with Eq. (19), where β_{ext} is the equilibrium reciprocal temperature fixed by X^s and E, while $H = X^s/\beta_{ext}$.

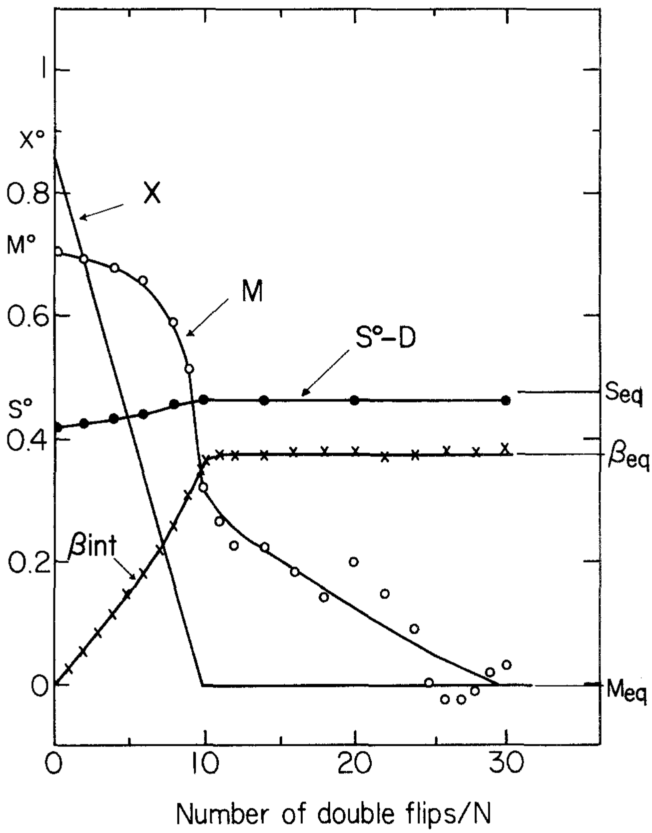

Fig. 3. Same as Fig. 1, but for an irreversible demagnetization in which X^s decreases from X^s to zero over the interval of $10N$ double-flips. $S^0 - D$ does not attain S_{eq} due to nonzero entropy production [Eq. (17)].

Figure 4 describes the most abrupt demagnetization, in which right from the beginning the magnetizing parameter X^s is put equal to zero ("switched off"). Since $D \equiv 0$ [Eq. (14)], the entire entropy of demagnetization is irreversibly lost and the entropy production [Eq. (17)] attains its maximum value

$$\phi = \Delta S_{\text{demag}} = 0.06 \tag{27}$$

But even for this most irreversible process the values of β_{int}, except for the very first ones, were found to obey Eq. (25) to a good approximation (for those that did not, β_{int} was calculated as an average for different l, l'). Furthermore, β_{int} cools to its equilibrium value $\beta_{eq} = 0.378$ fairly rapidly, after about N double-flips, while the magnetization attains $M_{eq} = 0$ only after $10N$ double-flips. Such findings support again the hypothesis of thermo-

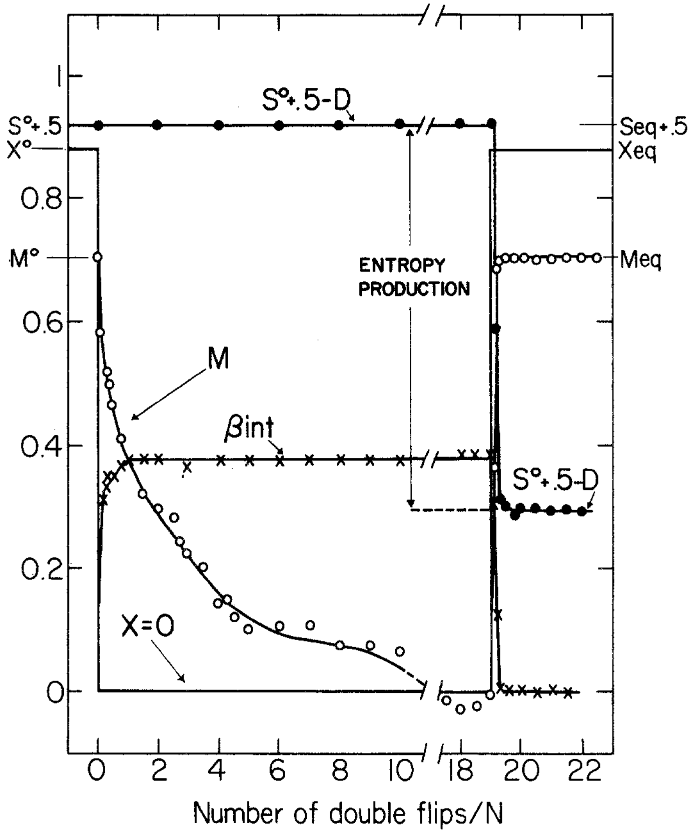

Fig. 4. Same as Figs. 1 and 3, but for the most irreversible demagnetization in which X^s is switched off (put to zero) right from beginning; hence $S^0 - D$ is nonincreasing. The original $X^s = X^0$ is switched on at time $19N$. The attendant remagnetization to M^0 and reheating to $\beta_{\text{int}} = 0$ are almost immediate; $S^0 - D$ falls to -0.2, corresponding to a very large entropy production.

dynamic equivalents for the process. The figure also describes an abrupt remagnetization of the lattice, obtained by switching on the original value of the magnetizing parameter $X^s = 0.881$ at time $s = 19N$. It is noted that the reestablishment of the initial magnetization M^0 and the attendant heating to $\beta_{\text{int}} = 0$ are almost instantaneous. This is to be expected since the adiabatic magnetization destroys the internal order which was established before by the demagnetization. The discrimination of the abrupt magnetization is very large, $D_{\text{mag}} = 0.62$ ($\simeq X^0 M^0$) and its entropy production is [Eq. (17)]

$$\phi = -\Delta S_{\text{demag}} + D_{\text{mag}} = 0.56 \tag{28}$$

This is much larger than the entropy production of the demagnetization

[Eq. (27)] and seems to agree with the observation just made regarding the relative rate of the two processes. The entropy production for the entire cycle $\phi = D_{\mathrm{mag}} = 0.62$ is indicated in the figure; it is obtained by adding Eqs. (27) and (28).

In conclusion, it may be said that the measurement of the discrimination for a model adiabatic process with no external work permits its entropy production to be evaluated without a priori recourse to thermodynamic equivalents. The results obtained lend a posteriori support to the use of such equivalents, even when the irreversibility of the process is quite pronounced.

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